

Fermionic van Hemmen Spin Glass Model with a Transverse Field

S. G. Magalhaes^a, F. M. Zimmer^b, C. V. Morais^b

^aInstituto de Física, Universidade Federal Fluminense, 24210-346 Niterói, RJ, Brazil

^bDepartamento de Física, Universidade Federal de Santa Maria, 97105-900 Santa Maria, RS, Brazil

Abstract

In the present work it is studied the fermionic van Hemmen model for the spin glass (SG) with a transverse magnetic field Γ . In this model, the spin operators are written as a bilinear combination of fermionic operators, which allows the analysis of the interplay between charge and spin fluctuations in the presence of a quantum spin flipping mechanism given by Γ . The problem is expressed in the fermionic path integral formalism. As results, magnetic phase diagrams of temperature versus the ferromagnetic interaction are obtained for several values of chemical potential μ and Γ . The Γ field suppresses the magnetic orders. The increase of μ alters the average occupation per site that affects the magnetic phases. For instance, the SG and the mixed SG+ferromagnetic phases are also suppressed by μ . In addition, μ can change the nature of the phase boundaries introducing a first order transition.

Keywords: quantum spin glass, fermionic model, charge fluctuation

It is well established that disorder can be a source of non-trivial effects in condensed matter physics. From Anderson localization to spin glass, there are plenty of examples supporting such role. As one more recent example there are indications that the presence of disorder is responsible by several complex effects in strongly correlated systems as, for instance, manganites or Cerium compounds (see Ref. [1] and references therein). The important point is that disorder can affect the charge sector as well as the spin one in such way that it becomes a constant cause of new interesting problems. From the theoretical point of view, the task of facing this kind of problems is surely a source of new approaches.

Recent experimental findings in two specific Cerium compounds can illustrate the last comment in the previous paragraph. $CePd_{1-x}Rh_x$ [2] and $CeNi_{1-x}Cu_x$ [3] are physical systems in which disorder combined with RKKY and Kondo interaction is responsible by the onset of a complex scenario. Their phase diagrams display a glassy state, ferromagnetism and a region dominated by the Kondo effect. For $CeNi_{1-x}Cu_x$, several works have used the so called Kondo-Ising lattice (KIL) model [4] to reproduce the experimental phase diagram of that compound. The Ising part of the model represents an intersite interaction between localized f-spin operators. It is precisely in the spin-spin Ising coupling J_{ij} that various types of randomness have been tested trying to represent the effects of the disorder in the $CeNi_{1-x}Cu_x$ [5, 6, 7, 8, 9]. For instance, the random coupling J_{ij} has been chosen as the Sherrington-Kirkpatrick (SK) model [10] or as the Hopfield model [11]. However, none of the choices previously mentioned were entirely satisfactory to describe how the glassy state in the $CeNi_{1-x}Cu_x$ is replaced by an inhomogeneous ferromagnetism when the temperature is decreased [3].

Very recently, a new approach has been proposed for the problem discussed above, this approach uses the random coupling J_{ij} as that one introduced in the van Hemmen (vH) model [12]. This model was originally conceived to study the spin glass (SG) problem with classical Ising spin variables. The results indicated that the vH type of randomness in the KIL model is more adequate than those ones used in SK or Hopfield models to reproduce some aspects of the experimental phase diagram of the $CeNi_{1-x}Cu_x$ [13]. For instance, the previously mentioned ferromagnetic phase appears below the spin glass one.

The improvement obtained using the coupling J_{ij} given as the vH model to study the $CeNi_{1-x}Cu_x$ suggests the usefulness of this kind of randomness could be more general. In other words, it could also be used when it is present in

Email address: ggarcia@ccne.ufsm.br (S. G. Magalhaes)

¹Tel.: +55 (055) 3220 9521

other quantum process than the Kondo effect. In this case, models with one type of fermions could be built by adding to the vH Ising term, for instance, a hopping, a pairing coupling [14] or a transverse magnetic field. Nevertheless, surprisingly little consideration has been given to a fermionic version of the vH model. In fact, even less consideration is given to the fermionic vH model when a term capable to introduce quantum dynamics in the problem is added to this model. In fact, it lacks an investigation of the interplay of charge and spin fluctuations within a fermionic version of the vH model even for a simple quantum process provided by a transverse magnetic field.

The purpose of the present work is to study the vH model in the presence of a transverse field Γ in a fermionic formulation which means that the Ising spin operators are written as bilinear combinations of creation and destruction fermionic operators. For this particular representation of the spin operators, the natural tool to obtain the thermodynamics is the functional integral formalism with Grassmann fields. A peculiarity of this representation is that \hat{S}_i^z acts in a space with four eigenstates. Among them, there are two which are nonmagnetic [15]. Therefore, we calculate the grand partition function for two situations. In the first one, here called 4S fermionic van Hemmen (4S-FvH) model, the four states are preserved. For this particular model, the chemical potential μ appears as an important parameter in the problem. In the second one, the 2S fermionic van Hemmen (2S-FvH) model, introduces a restriction, what allows considering only the two magnetic states [15, 16]. One of the most important advantages of the vH model in both versions 4S and 2S is that the disorder can be treated without the use of the replica method. Therefore, the subsequent difficulties associated with this technique [17] can be avoided in the present work (see discussion below). However, the problem is still treated within the static approximation (SA) in which the time fluctuations are neglected [18].

Indeed, the fermionic representation of Ising spin operators for the study of the SG problem is well established. For the SK type of randomness, the so called fermionic Ising spin glass (FISG) model [16, 19, 20] shows an interesting characteristic. Charge and spin correlation functions are connected. The average occupation of fermions per site and the replica diagonal element of the SG order parameter matrix are directly related. This relationship implies that, for instance, the nature of the PM/SG phase boundary can be modified by variations of chemical potential with the onset of a tricritical point [21]. The presence of an additional transverse magnetic field Γ in the FISG model also introduces important changes in the PM/SG phase boundary. The first order part of that phase boundary is diminished when Γ increases [22]. Consequently, the position of the tricritical point is also affected. Therefore, one could expect that in the 4S-FvH model, variations of μ could also bring important consequences on the location and nature of phase boundaries.

It should be remarked that the location of a first order PM/SG phase boundary is not a trivial task for models in which the replica method is used as the FISG one [22, 23]. This difficulty has already been a subject of controversy for the classical Ghatak-Sherrington model (see, for instance, Refs. [24, 25]). It is clear that this kind of controversy would not be needed in the FvH model. On the other hand, vH model (classical or fermionic) has a shortcoming. It lacks to present a multiplicity of metastable-states [26]. Nevertheless, the vH model produces some results which are close to those ones found experimentally in SG physical systems (see Ref. [27]). Thus, the proper understanding of the vH model in a fermionic formulation can be important not only as a SG problem itself but also as something useful for applications as pointed out above and very recently discussed in Ref. [13].

It should also be mentioned that the vH model with an additional transverse magnetic field Γ has already been studied. However, in a formulation where the classical Ising spin variables were replaced by Pauli matrices [28]. In this formulation one can observe important consequences for the phase diagram produced by the presence of Γ . The obtained phase diagram shows that the increase of Γ destroys the SG, the ferromagnetic (FM) and the mixed (FM+SG) phases found in the classical phase diagram. Nevertheless, this kind of formulation is not able to capture the consequences for the phase diagram when both charge and spin fluctuations are affected by Γ .

This paper has the following structure. In the second section, the model is introduced and the grand canonical potential is derived. In the third section, several phase diagrams are constructed with particular attention for the effects of Γ and μ by solving the equations for the order parameters. The last section is dedicated to the conclusions.

1. Model

The Hamiltonian of the fermionic van Hemmen (FvH) model with a magnetic transverse field Γ is given by

$$H = -\frac{2J_0}{N} \sum_{i \neq j} \hat{S}_i^z \hat{S}_j^z - 2 \sum_{i \neq j} J_{ij} \hat{S}_i^z \hat{S}_j^z - 2\Gamma \sum_i \hat{S}_i^x \quad (1)$$

with the operators

$$\hat{S}_i^z = \frac{1}{2}[c_{i\uparrow}^\dagger c_{i\uparrow} - c_{i\downarrow}^\dagger c_{i\downarrow}], \quad \hat{S}_i^x = \frac{1}{2}[c_{i\uparrow}^\dagger c_{i\downarrow} + c_{i\downarrow}^\dagger c_{i\uparrow}] \quad (2)$$

and the random coupling J_{ij} given by

$$J_{ij} = \frac{J}{N}[\xi_i \eta_j + \xi_j \eta_i], \quad (3)$$

where ξ_i and η_i are random variables which follow a bimodal distribution

$$P(\xi_i) = \frac{1}{2}[\delta(\xi_i - 1) + \delta(\xi_i + 1)]. \quad (4)$$

In the present work the partition function is obtained within the Lagrangian path integral formalism. In that case, the spin operators are represented as bilinear combinations of Grassmann fields (φ, φ^*). Particularly, the 2S-FvH model admits only magnetic states. This restriction for the corresponding partition function is obtained by using the Kronecker δ function ($\delta(\hat{n}_{i\uparrow} + \hat{n}_{i\downarrow} - 1) = \frac{1}{2\pi} \int_0^{2\pi} dx_i e^{ix_i(\hat{n}_{i\uparrow} + \hat{n}_{i\downarrow} - 1)}$) [5, 29]. This procedure allows to write the partition function for 2S and 4S situations in a compact form as

$$Z\{y\} = e^{\frac{s-2}{2}\beta\mu} \prod_j \frac{1}{2\pi} \int_0^{2\pi} dx_j e^{-y_j} Z_{eff}\{y\} \quad (5)$$

and

$$Z_{eff}\{y\} = \int D(\varphi^* \varphi) e^{A\{y_j\}} \quad (6)$$

where s ($s = 2$ or 4) is the number of states per site,

$$A\{y_j\} = \int_0^\beta d\tau \left\{ \sum_{j,\sigma} \varphi_{j\sigma}^*(\tau) \left[-\frac{\partial}{\partial \tau} + \frac{y_j}{\beta} \right] \varphi_{j\sigma}(\tau) - H(\varphi_{j\sigma}^*(\tau), \varphi_{j\sigma}(\tau)) \right\}, \quad (7)$$

μ is chemical potential, $\beta = 1/T$ and $y_j = \beta\mu$ or $y_j = ix_j$ for the 4S-FvH and 2S-FvH models, respectively.

The action $A\{y\}$ given in Eq. (6) can be build using the hamiltonian (1). Thus, we have:

$$A\{y_j\} = A_\Gamma + A_{SG} + A_{FE} \quad (8)$$

where

$$A_\Gamma = \int_0^\beta \sum_j \underline{\varphi}_j^\dagger(\tau) \left[\frac{y_j}{\beta} - \frac{\partial}{\partial \tau} + \Gamma \underline{\sigma}_x \right] \underline{\varphi}_j(\tau), \quad (9)$$

$$A_{SG} = \int_0^\beta \sum_{(i,j)} \frac{J_{ij}}{2} S_i(\tau) S_j(\tau), \quad (10)$$

$$A_{FE} = \frac{J_0}{N} \int_0^\beta d\tau \sum_{(i,j)} S_i(\tau) S_j(\tau) \quad (11)$$

and

$$S_i(\tau) = \underline{\varphi}_i^\dagger(\tau) \underline{\sigma}^z \underline{\varphi}_i(\tau). \quad (12)$$

The matrices in Eqs. (9)-(12) are defined as:

$$\underline{\varphi}_i(\tau) = \begin{bmatrix} \varphi_{i\uparrow}(\tau) \\ \varphi_{i\downarrow}(\tau) \end{bmatrix}; \quad \underline{\sigma}^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \underline{\sigma}^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (13)$$

The coupling J_{ij} given in Eq. (3) allows to rewrite the random part of the action as:

$$A_{SG} = \frac{J}{2N} \int_0^\beta d\tau \left\{ \left[\sum_{j=1}^N (\eta_j + \xi_j) S_j(\tau) \right]^2 - \left[\sum_{j=1}^N \eta_j S_j(\tau) \right]^2 - \left[\sum_{j=1}^N \xi_j S_j(\tau) \right]^2 - 2 \sum_{j=1}^N \eta_j \xi_j (S_j(\tau))^2 \right\}. \quad (14)$$

Similarly, the ferromagnetic part of the action is given as

$$A_{FE} = \frac{J_0}{2N} \int_0^\beta d\tau \left\{ \left[\sum_j S_j(\tau) \right]^2 - \sum_j [S_j(\tau)]^2 \right\}. \quad (15)$$

The Hubbard-Stratonovich transformation is used to linearize the first three terms and the first one from the right side of Eqs. (14) and (15), respectively. The last terms in the right side of Eqs. (14)-(15) vanish in the thermodynamic limit. Therefore, $Z\{y\}$ defined in Eqs. (5)-(12) is obtained as:

$$Z\{y\} = e^{\frac{\beta-2}{2}\beta\mu} \int Dq_1(\tau) \int Dq_2(\tau) \int Dq_3(\tau) \int Dm(\tau) \times \exp\left\{-N\left[\frac{1}{2} \int_0^\beta d\tau (q_1(\tau)^2 + q_2(\tau)^2 + q_3(\tau)^2 + m(\tau)^2) - \frac{1}{N} \ln \left(\int_0^{2\pi} \prod_j \frac{dx_j}{2\pi} e^{-iy_j} \Lambda(q_1, q_2, q_3, m) \right) \right]\right\} \quad (16)$$

with

$$\Lambda(q_1, q_2, q_3, m) = \int D(\varphi^* \varphi) \times \exp\left\{ \sum_j \int_0^\beta d\tau \varphi_j^\dagger(\tau) \underline{G}^{-1}(\tau|h_j) \varphi_j(\tau) \right\} \quad (17)$$

and

$$\underline{G}(\tau|h_j) = \left[\frac{y_j}{\beta} - \frac{\partial}{\partial \tau} + h_j \underline{\sigma}^z + \beta \Gamma \underline{\sigma}^x \right]. \quad (18)$$

In Eq. (16), it has been used the self-averaging property $\frac{1}{N} \sum_j f(\xi_j; \eta_j)$. The average $\langle \dots \rangle_{\xi, \eta}$ can be achieved by using the probability distribution $P(\xi, \eta) = P(\xi)P(\eta)$ where $P(\xi)$ (or $P(\eta)$) is defined in Eq. (4). In this work, the SA is assumed, in which $m = m(\tau)$ and $q_n = q_n(\tau)$ for $n=1, 2$ and 3 . Therefore, the internal field in Eq.(18) is

$$h_j = \sqrt{J} (i\eta_j q_1 + i\xi_j q_2 + (\eta_j + \xi_j) q_3) + \sqrt{J_0} m. \quad (19)$$

The integrals over $\{q_n\}$ and m in Eq. (16) can be solved by the saddle point method which gives

$$q_1 = i \sqrt{J} \frac{1}{N} \sum_j \langle \xi_j S_j \rangle = i \sqrt{J} Q_1 \quad (20)$$

$$q_2 = i \sqrt{J} \frac{1}{N} \sum_j \langle \eta_j S_j \rangle = i \sqrt{J} Q_2 \quad (21)$$

$q_3 = Q_1 + Q_2$ and

$$m = \sqrt{J_0} \frac{1}{N} \sum_j \langle S_j \rangle = \sqrt{J_0} M. \quad (22)$$

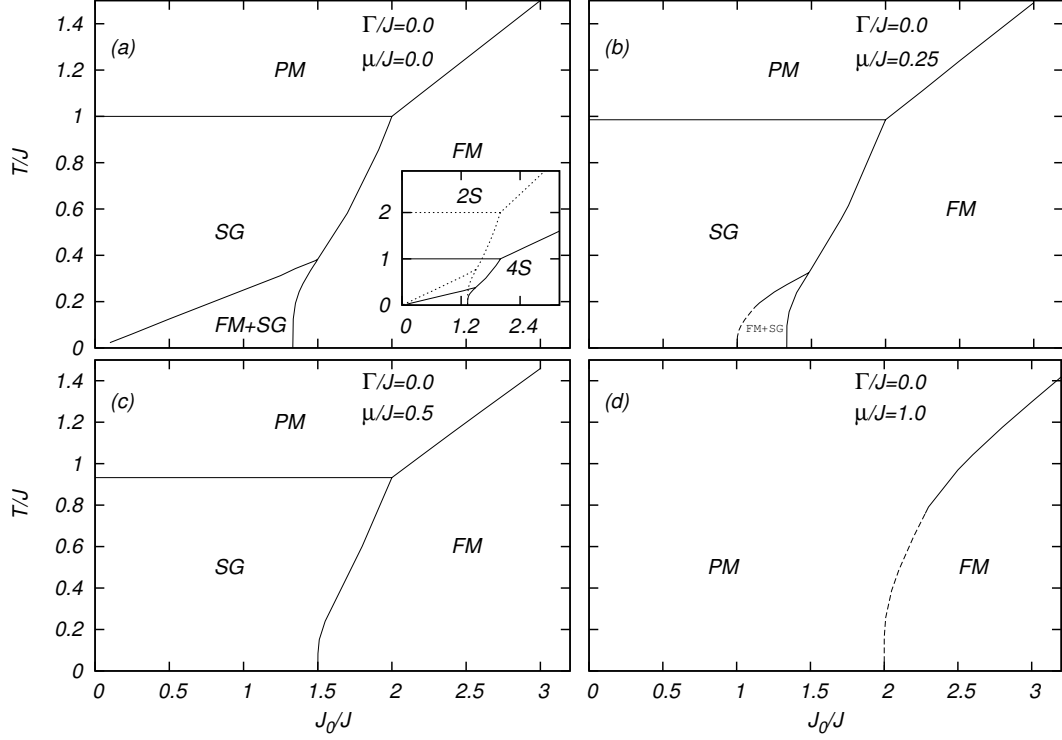


Figure 1: Phase diagrams T/J versus J_0/J for several values of μ/J and $\Gamma/J = 0$. The solid lines correspond to the second order transition while the dashed lines indicate first order transition. The inset shows a comparison between the phase diagrams for the 2S-fvH model (dotted lines) and the 4S-fvH model with $\mu = 0$.

The Fourier transform can be used in Eq. (17) and the functional integrals over the Grassmann fields (φ, φ^*) performed as well as the subsequent sum over Matsubara's frequencies following close Ref. [15] to give:

$$\Lambda(Q_1, Q_2, M) = 2e^y [\cosh(y) + \cosh(\sqrt{\beta\Delta})] \quad (23)$$

with $\Delta = [J(\eta + \xi)(Q_1 + Q_2) - J\eta Q_1 - J\xi Q_2 + J_0 M]^2 + \Gamma^2$.

For the 2S-FvH model the integral over x in Eq. (16) is calculated which leads to the same thermodynamics found in Ref. [28] for the quantum vH model. While, in the case of 4S-FvH model, $Z\{\mu\}$ can also be obtained from Eq.(16) which gives for the grand canonical potential Ω , the following expression:

$$\begin{aligned} \beta\Omega &= \beta J Q^2 + \frac{\beta J_0}{2} M^2 - \beta\mu \\ &\quad - \langle \ln 2 [\cosh(\beta\mu) + \cosh(\beta\sqrt{\Delta})] \rangle_{\eta\xi} \end{aligned} \quad (24)$$

where it is assumed $Q_1 = Q_2 = Q$.

2. Results

In order to investigate the interplay between charge and spin fluctuations in the 4S-FvH model in the presence of a transverse magnetic field Γ , a set of phase diagrams can be built from the numerical solution of the saddle point SG order parameter Q and the magnetization M which is obtained from Eq. (24). It should be remarked that temperature, J_0 (the strength of the ferromagnetic component of J_{ij}), μ and Γ are given in units of J (the strength of the random component of J_{ij}). For numerical results, $J = 2$ is used. In Figs. 1-4, phase diagrams temperature versus J_0 for several values of μ and Γ are displayed.

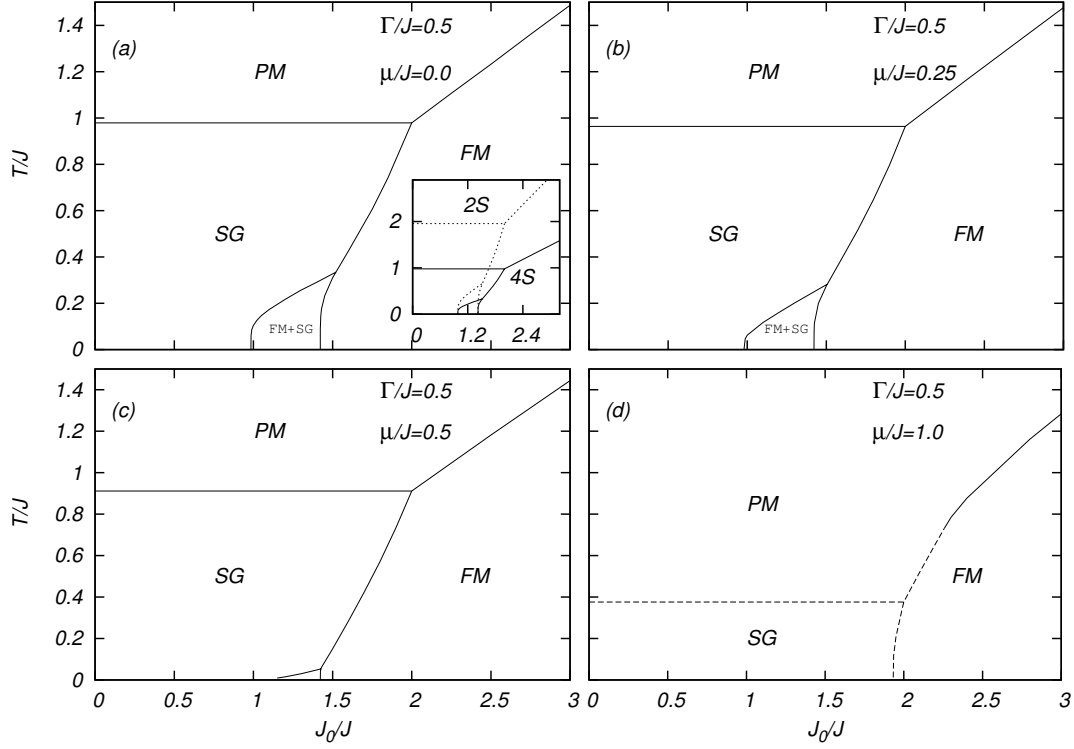


Figure 2: Phase diagrams T/J versus J_0/J for several values of μ/J and $\Gamma/J = 0$. The solid lines correspond to the second order transition while the dashed lines indicate first order transition. The inset shows a comparison between the phase diagrams for the 2S-FvH model (dotted lines) and the 4S-FvH model with $\mu = 0$.

Fig. 1(a) shows the situation corresponding to the half-filling occupation ($\mu = 0$) and $\Gamma = 0$. For that case, the phase diagram of the classical vH model is basically recovered. For small J_0 , when the temperature is decreased, there is first a transition from the PM phase to the SG one which has $M = 0$ and $Q \neq 0$. Then, there is a second transition from the SG phase to the mixed (FM+SG) one in which $M \neq 0$ and $Q \neq 0$. For larger J_0 , SG and FM+SG phases are replaced by the ferromagnetic (FM) one with $M \neq 0$ and $Q = 0$. In the inset of Fig. 1(a), the 4S-FvH model phase diagram with $\mu = 0$ is compared to the 2S-FvH one. These two models present qualitatively the same results. However, the phase boundaries of the 4S-FvH model at the half-filling appear at lower temperatures than the 2S-FvH model. That is a consequence of the non-magnetic states allowed in the 4S-FvH model. Figs. 1(b)-1(d) show how the increase of μ can affect phase boundaries even when $\Gamma = 0$. When the average occupation n moves from the half-filling ($\mu = 0$) to doubly occupied sites ($\mu = 1$), the transition temperature T_f between paramagnetism and SG phase as well as that one (here called T_g) between SG and FM+SG phases are depressed. Ultimately, this depressing of T_f and T_g as μ increases leads to the complete suppression of SG and FM+SG. Similarly to the FISG model (see discussion in Ref. [21]), the increase of μ also changes the nature of the phase transitions which appear in Fig. 1. That can be seen, for instance, for T_g in Fig. 1(b) when $\mu = 0.25$. Furthermore, for $\mu = 1$ (see Fig. 1(d)), the nature of the Curie temperature T_c is also changed. It appears a tricritical point which can be located (see Appendix) at $T_c^{tric} = 0.759$ with $J_0^{tric} = 2.278$. One last consequence due to the increase of μ is that larger values of J_0 are necessary to obtain a FM solution.

In Fig 2, $\Gamma = 0.5$ while μ assumes the same values already used in Fig. 1. Therefore, the results can be compared directly to Fig. 1 which allows to understand the combined effects of Γ and μ . The phase diagram for $\mu = 0$ is shown in Fig. 2(a). In this case, the effect of Γ is to depress both T_f and T_g . However, the subsequent increase of μ showed in Figs. 2(b)-2(d) leads to a distinct situation as compared with Fig 1. For instance, the FM+SG phase is completely suppressed. In contrast, the SG phase is still preserved. Other important difference in comparison to the case displayed in Fig. 1 is concerned with the nature of phase boundaries. For instance, the line transition T_g remains always a second order one. That is not the case for T_f which becomes a first order one as displayed in Fig.

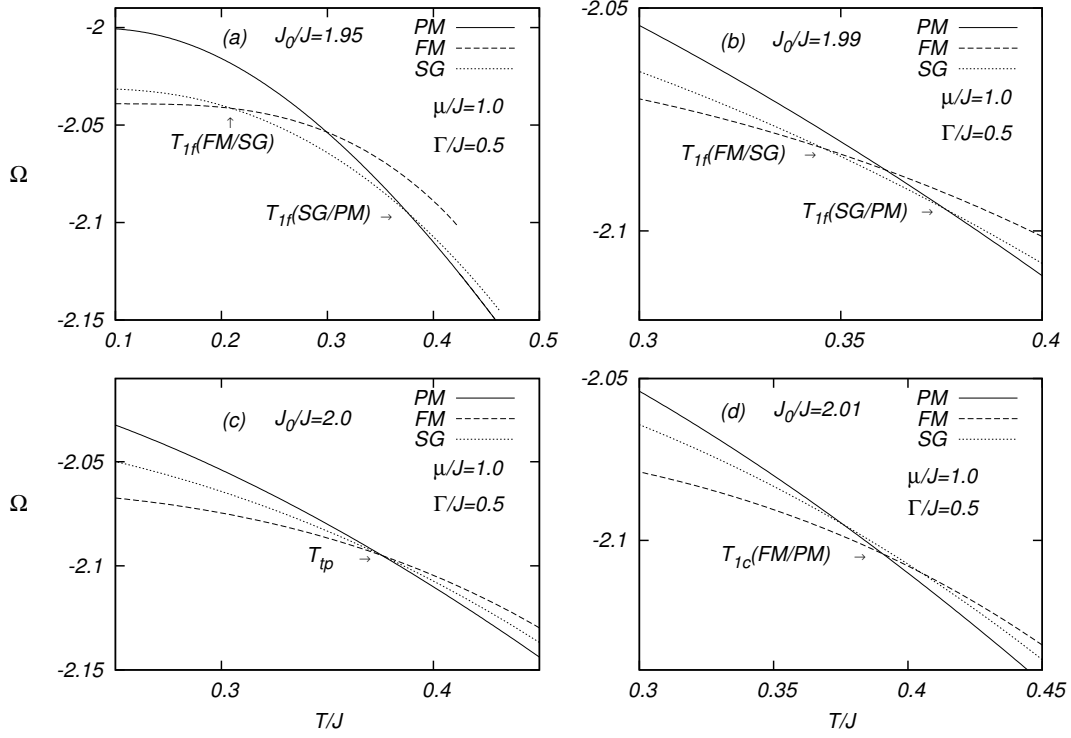


Figure 3: Grand canonical potential Ω versus T/J for $\mu/J = 1.0$, $\Gamma/J = 0.5$ and several values of J_0 .

2(d) where $\mu = 1$. In fact, for this value of chemical potential there is a completely new scenario. The existence of the tricritical point in the Curie temperature is preserved being located at $T_c^{tric} = 0.729$. Nevertheless, there is now an additional triple point $T_{triple} \approx 0.376$ with the coexistence of SG, FM and PM phases. Fig. 3 exhibits the grand canonical potential Ω as a function of temperature for the SG, PM and FE phases near the triple point. The first order transitions are located by comparing Ω of these phases, where the stable solution is always that one with minimum Ω . Particularly, the triple point is obtained when Ω of the SG, PM and FE phases assume the same values (see Fig. 3(c))

In Fig. 4 phase diagrams with $\Gamma = 1.0$ are displayed. In that case, the FM+SG is completely suppressed for $\mu = 0$ (see Fig. 4(a)). The remaining phase diagrams displayed in Figs. 4(b) -4(d) present only SG, FM and PM solutions for the order parameters. When $\mu = 0.25$ and 0.5 , the second order line transition T_f is even more diminished when compared to the respective previous situations given in Figs. 1-2. For $\mu = 1$, the complex scenario found in Fig. 2(d) is repeated in Fig. 4(d) with the presence of tricritical and triple points. However, interestingly, T_f , which is now a first order line transition, has a small increase as compared with Fig 2(d).

The effects of the Γ increasing on the 2S-FvH model can be observed in the inset of Figs. 2(a) and 4(a), which exhibit phase diagrams of the 2S-FvH model compared to the 4S one with $\mu = 0$ for $\Gamma = 0.5$ and 1.0 , respectively. As in the 4S-FvH model, the magnetic phase is also suppressed by Γ in the 2S-FvH model. In addition, the phase boundaries converge to the same values when temperature decreases.

3. Conclusion

The present work has studied the fermionic version of the classical van Hemmen model [12] for SG in the presence of a transverse magnetic field Γ . The goal is to verify how the interplay between charge and spin fluctuations can affect the magnetic ordering in this particular model. It should be mentioned that the knowledge of such interplay can also be quite useful in further applications for studying the disordered magnetism when other physical processes are present (see, for instance, Ref. [13]).

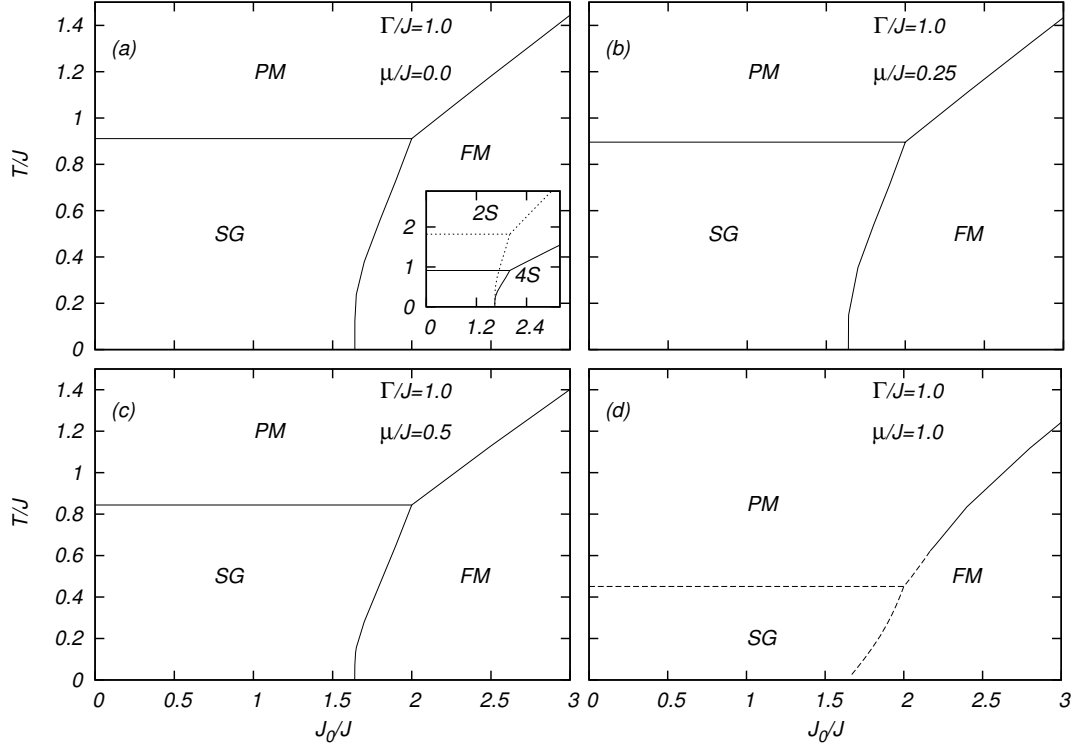


Figure 4: Phase diagrams T/J versus J_0/J for several values of μ/J and $\Gamma/J = 1.0$. Dashed lines indicate first order transition. The inset shows phase diagrams for the 2S-fvH model (dotted lines) and the 4S-fvH model with $\mu = 0$ and $\Gamma = 1$.

The mentioned fermionic version has been formulated in two versions. In the first one, the so called 2S-FvH, the two nonmagnetic eigenstates of the operator \hat{S}_i^z (defined in Eq. (12) have been eliminated. Therefore, the corresponding thermodynamics obtained is exactly the same one obtained by a previous quantum formulation of the van Hemmen model given in Ref. [28]. On the contrary, when the four original eigenstates are preserved, the solution for both saddle point order parameters, the SG (Q) and the magnetization (M) one, indicates that not only the phase diagram but also the nature of the phase transitions are deeply affected when the chemical potential μ and/or Γ are varied. For instance, the mixed phase FM+SG can be suppressed as well as there is onset of tricritical and triple points by increasing μ and/or Γ . In fact, the role of Γ as responsible for the suppression of the FM+SG phase has already been found in Ref. [28]. Remarkably, in the present problem, the same result can be obtained by varying only μ , or in other words, only by changing the average occupation of sites n . Therefore, one can say that the thermodynamics derived from van Hemmen model is either sensitive to a spin flipping mechanism provided by Γ as well a dilution one provided by μ . Surely, when both mechanisms are combined, the complexity of the phase diagram is even more intense as can be illustrated by a first order phase transition between the PM and SG (or FE) phases with the presence of tricritical and triple points.

To conclude, we have shown in this work that mechanisms such as dilution and quantum spin flipping given by μ and Γ , respectively, are sources of quite non-trivial effects in the fermionic van Hemmen model. More precisely, those effects indicate that the existence of magnetic phases and the nature of its boundaries in that model are extremely sensitive to the redistribution of charge provided by μ and Γ . It should be emphasized that thermodynamics in the present work could be derived without the use of sophisticated mathematical techniques such as the replica method. It is well known that in the case of SG first order phase transitions, the use of this method implies in considerable difficulties as, for example, location of first order phases boundary [22, 23, 24, 25, 30, 31].

Acknowledgments

This work was partially supported by the Brazilian agencies CNPq and CAPES.

Appendix A.

The Landau expansion of the grand canonical potential (Eq. (24)) in powers of M and Q is given by

$$\beta\Omega = \Omega_0 + A_2 Q^2 + \frac{\beta^2 J^4 A}{\Gamma^2} Q^4 + B_2 M^2 + \frac{\beta^2 J_0^4 A}{8\Gamma^2} M^4 \quad (\text{A.1})$$

where

$$A_2 = \beta J \left(1 - \frac{J \sinh \beta \Gamma}{\Gamma K_0}\right), \quad B_2 = \frac{\beta J_0}{2} \left(1 - \frac{J_0 \sinh \beta \Gamma}{\Gamma K_0}\right), \quad (\text{A.2})$$

$$A = -\frac{1 + \cosh \beta \Gamma \cosh \beta \mu}{K_0^2} + \frac{\sinh \beta \Gamma}{\beta \Gamma K_0} \quad (\text{A.3})$$

and

$$K_0 = \cosh \beta \mu + \cosh \beta \Gamma. \quad (\text{A.4})$$

The second order phase transition from the paramagnetic phase to the SG one (to the FE phase) occurs when $A_2 = 0$ ($B_2 = 0$) and $A > 0$. The tricritical point is located when $B_2 = 0$ with $A = 0$.

References

- [1] E. Dagotto, Science 309 (2005) 257-262.
- [2] M. Brando, J. Hemberger, T. Westerkamp, M. Deppe, R. Kuchler, M. Brando, C. Geibel, P. Gegenwart, A. P. Pikul, F. Steglich, Phys. Rev. Lett. 102 (2009) 206404.
- [3] N. Marcano, J. C. Gomez Sal, J. I. Espeso, J. M. De Teresa, P. A. Algarabel, C. Paulsen, J. R. Iglesias, Phys. Rev. Lett. 98 (2007) 166406.
- [4] B. Coqblin, M. D. Nunez-Regueiro, A. Theumann, J. R. Iglesias, S. G. Magalhaes, Philosophical Magazine 86 (2006) 2576-2580.
- [5] A. Theumann, B. Coqblin, S. G. Magalhaes, A. A. Schmidt, Phys. Rev. B 63 (2001) 054409.
- [6] S. G. Magalhaes, A. A. Schmidt, A. Theumann, B. Coqblin, Eur. Phys. J. B 30 (2002) 419-426.
- [7] A. Theumann, B. Coqblin, Phys. Rev. B 69 (2004) 214418.
- [8] S. G. Magalhaes, F. M. Zimmer, P. R. Krebs, B. Coqblin, Phys. Rev. B 74 (2006) 014427.
- [9] S. G. Magalhaes, F. M. Zimmer, B. Coqblin Physica B 403/5 (2008) 1395-1397.
- [10] S. Kirkpatrick, D. Sherrington, Phys. Rev. B 17 (1978) 4384-4403.
- [11] D. J. Amit, Modelling Brain Function. The world of Attractor Neural Networks, Cambridge University Press, Cambridge, England, 1989.
- [12] J. L. van Hemmen, Phys. Rev. Lett. 49 (1982) 409-412.
- [13] S. G. Magalhaes, F. M. Zimmer, B. Coqblin, Phys. Rev. B 81 (2010) 094424.
- [14] S. G. Magalhaes, A. Theumann, Eur. Phys. J. B vol 9 (1999) 5-14.
- [15] A. Theumann, A. A. Schmidt, S. G. Magalhaes, Physica A 311 (2002) 498-506.
- [16] W. Wiethege, D. Sherrington, J. Phys. C 19 (1986) 6983-6991.
- [17] K. Binder, A. P. Young, Rev. Mod. Phys. 58 (1986) 801-976.
- [18] A. J. Bray, M. A. Moore, J. Phys. C 13 (1980) L655-L660.
- [19] A. Theumann, M. V. Gusmao, Phys. Lett. A 105 (1984) 311-314.
- [20] R. Oppermann, A. Muller-Groeling, Nucl. Phys. B 401 (1993) 507-547.
- [21] B. Rosenow, R. Oppermann, Phys. Rev. Lett. 77 (1996) 1608-1611.
- [22] S. G. Magalhaes, C. V. Morais, F. M. Zimmer, Phys. Rev. B 77 (2008) 134422.
- [23] S. G. Magalhaes, C. V. Morais, F. M. Zimmer, Phys. Rev. B 81 (2010) 014207.
- [24] S. K. Ghatak, D. Sherrington, J. Phys. C 10 (1977) 3149-3156.
- [25] F. A. da Costa, C. S. O. Yokoi, S. R. A. Salinas, J. Phys. A 27 (1994) 3365-3372.
- [26] T. C. Choy, D. Sherrington, J. Phys. C: Solid State Phys. 17 (1984) 739-745.
- [27] J. L. van Hemmen, J. Phys. C: Solid State Phys. 19 (1986) L379-L382.
- [28] J. Roberto Viana, Yamiles Nogueira, J. Ricardo de Souza, Phys. Rev. B 66 (2002) 113307.
- [29] S. G. Magalhaes, F. M. Zimmer, C. V. Morais, Physica A 388 (2009) 2140-2148.
- [30] P. Mottishaw, D. Sherrington, J. Phys. C 18 (1985) 5201-5213.
- [31] P. Mottishaw, Europhys. Lett. 1 (1986) 409-414.